SHM Worksheet

(1) - yes, the movement is periodic and moves back and forth around an equilibrium position (middle of the walls)
- yes it is SHM; acceleration is in the opposite direction of the motion, acceleration is proportional to displacement, the motion is sinusoidal in nature.

(2) (a) 5.0 mm

(b) $y = 5.0\cos(2(1.2s)) = \underline{-3.7\text{ mm}}$

(c) $-2.0\text{ mm} = 5.0\cos(2t)$

$$t = \frac{\cos^{-1}\left(\frac{-2.0m}{5.0}\right)}{2} = \underline{0.99s}$$

(d) $v = \pm\omega\sqrt{x_0^2 - x^2}$

$$x = \pm\sqrt{x_0^2 - \left(\frac{v}{\omega}\right)^2} = \pm\sqrt{(5.0)^2 - \left(\frac{6\text{ mms}^{-1}}{2}\right)^2} = \underline{\pm 4.0\text{ mm}}$$

(3) $\omega = 2\pi f = 2\pi(14\text{ Hz}) = 28\pi$

$x = 8.0\cos(28\pi t)$

(4) $v = \pm\omega\sqrt{x_0^2 - x^2}$ for maximum v, $x = 0$

$\omega = 2\pi f$

$= \pm 2\pi(460\text{ Hz})\sqrt{(5\times10^{-3}m)^2}$

$\underline{v = 14\text{ ms}^{-1}}$

$a = -\omega^2 x$

$= -(2\pi\, 460\text{ Hz})^2(5\times10^{-3}m)$

$\underline{a = 4.2\times10^4\text{ ms}^{-2}}$

(5) $f = 4500\ rpm = 75\ Hz$

$\omega = 2\pi f = 2\pi(75\ Hz) = 471.24\ s^{-1}$

(a) $a = -\omega^2 x$

$= -(471.24\ s^{-1})^2 (0.045\ m)$

$a = \underline{1 \times 10^4\ ms^{-2}}$

maximum displacement would be half the total distance.

(b) $v = \pm \omega \sqrt{x_0^2 - x^2}$ at equilibrium point $x = 0$

$v = \pm (471.24)(0.045\ m)$

$v = \underline{21\ ms^{-1}}$

(c) $F = ma$

$= (0.25\ kg)(1 \times 10^4\ ms^{-2}) = \underline{2500\ N}$

(6) $v = \pm \omega \sqrt{x_0^2 - x^2}$

$v_0 = \pm \omega \sqrt{A^2 - x_0^2}$

Amplitude will occur when $v = v_0$ and $x = x_0$; x_0 in the equation is the amplitude. so rewrite as A.

$\left(\frac{v_0}{\omega}\right)^2 = A^2 - x_0^2$

$A = \sqrt{x_0^2 + \frac{v_0^2}{\omega^2}}$

(7) (a) As the object moves in the $+x$ direction, acceleration increases in the negative direction indicating a force acting in the $-x$ direction pulling the object back to zero.
Similarly when the object moves in the $-x$ direction there is a force acting in the $+x$ direction pulling the object back to the equilibrium position $(x=0)$
The object will constantly oscillate

7 (b) Slope of graph is $-\omega^2$

$$\frac{rise}{run} = \frac{^-1.5 - 1.5}{.10 - ^-.10} = \frac{^-3}{.20} = ^-15 \ s^{-2}$$

$$\omega = \frac{2\pi}{T} \qquad -\omega^2 = -15 \ s^{-2}$$

$$\omega = 3.87 \ s^{-1}$$

$$T = \frac{2\pi}{\omega} = \frac{2\pi}{3.87 s^{-1}} = \underline{1.62 \ s}$$

(c) $v = \pm \omega \sqrt{x_0^2 - x^2}$ max $x = 0$

$$v = (3.87 s^{-1})(.1 m) = \underline{0.387 \ ms^{-1}}$$

(d) $F = ma$ $\quad a = -\omega^2 x$

$$F = -m\omega^2 x$$

$$= -(0.150 kg)(15 s^{-2})(0.1) = \underline{0.23 N}$$

(e) $E_T = \frac{1}{2} m\omega^2 x_0^2 = \frac{1}{2}(.150 kg)(15 s^{-2})(0.1 m)^2$

$$\underline{E_T = 0.011 \ J}$$

8 (a) amplitude $= \underline{0.360 \ m}$

frequency $= \frac{\omega}{2\pi} = \frac{6.80}{2\pi} = \underline{1.08 \ Hz}$

period $= \frac{1}{f} = \underline{0.924 \ s}$

(b) $E_T = \frac{1}{2} m\omega^2 x_0^2 = \frac{1}{2}(1.80 kg)(6.80 s^{-1})^2 (0.360 m)^2$

$$= \underline{5.39 \ J}$$

8 (c) $E_k = \frac{1}{2} m\omega^2 (x_0^2 - x^2)$

$= \frac{1}{2}(1.80\,kg)(6.80\,s^{-1})^2(0.360\,m^2 - 0.125\,m^2)$

$= \underline{4.74\ J}$

$E_T = E_k + E_p$

$E_p = E_T - E_k = 5.39\,J - 4.74\,J = \underline{0.650\ J}$

9 - we do not want the car to oscillate
- we want the shock absorber to recover quickly

Therefore, it should be critically damped.

10 - Marching in step is a external force with a frequency.
- if the frequency of the marching matches the natural frequency of the bridge, then resonance will occur
- a resonating bridge could collapse